

Relationship between Mathematical Thinking and Mathematics Achievement

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Abstract: *In Nepalese context or in our experience, Mathematics is taken as a dry subject at school level and therefore students' achievement in mathematics has always been burning issue among the stakeholders as the level appears lower than expectation. So if it is desired to improve Mathematics Achievement, measures have to be taken to improve Mathematical Thinking. The study was carried out in order to find the relationship between Mathematical Thinking and Mathematics Achievement of the grade X students in Nepal. Three districts were selected for study area Sindhupalchok, Kathmandu and Mahottari according to the geographical figure i.e. From the Mountain, the Hill & Valley and the Terai region respectively. 400 students were selected from the study area through stratified random sampling. They were selected in different strata like: gender, location and sector wise. Two types of test were developed by the researcher. The researcher tried to find the result using Pearson Correlation Coefficient using SPSS 20.00 version. Finally, the researcher got the result: the correlation coefficient of Mathematical Thinking with Mathematics Achievement is $r = 0.731$ which significant at $p < 0.01$. There is strongly relationship between Mathematical Thinking and Mathematics Achievements.*

Keywords: *Mathematics Achievement, Mathematical Thinking, Relationship*

I. Introduction

There is a huge variety of notation on meaning of Mathematical thinking. In the study of, Karadag, Z. (2009) stated that mathematical thinking is a thinking style supported by thinking skills. This definition is not emphasized on the attitudes of the students. So, it should be defined in another way. Mason, J.; Burton, L & Stacy, K. stated "Mathematical thinking is a dynamic process which, by enabling us to increase the complexity of ideas we can handle, expands our understanding" (1991). The development of mathematical thinking plays an important role on the solution of problems faced in daily life (Pinar Bal & Ahmet, 2014). It showed Mathematical thinking is a part of our life. C.S. Lim, C. S. & T.Y. defined mathematical thinking as a mental operation used in solving problem affirmed by the mathematical knowledge and dispositions (2006). Furthermore, the educators developed the definition of mathematical thinking based on their own country's curriculum (Isoda, 2006).

Mathematical thinking is a broad term containing many perspectives and meanings. Each researcher dealing with mathematics education has his or her own perspective on this topic. Most researchers, mathematicians and mathematics educators define mathematical thinking as a process, which contains at least one of the mental and math-related activities such as reasoning, abstracting, conjecturing, representing and switching between different representations, visualizing, deducing, inducing, analyzing, synthesizing, connecting, generalizing, and proving (Schoenfeld, 1992; Tall, 1991; Burton, 1984). In this study, Mathematical thinking is defined as "the development of a mathematical point of view- valuing the process of mathematization and abstraction and having the predilection to apply them; and the development of competence with tools of the trade and using those tools in the service of the goal of understanding structure" (Schoenfeld, 1992, p. 335)

Student achievement is seen as the most important indicator of national development. A country cannot develop its socioeconomic situation without enhancing students' achievement (Greaney & Kellaghan, 2008b) focused to increase student achievement that is the key to alleviating poverty and improving economic competitiveness. In the Standards for test construction achievement is viewed basically as the competence a person have in an area of content. This competence is the result of many intellectual and nonintellectual variables. The Mathematics Achievement is seemed as the marks obtained on mathematics subjects by the students.

(Ma'Moon, 2005) had studied to examine relationships between mathematical thinking and Mathematics Achievement through the use of both quantitative and qualitative data. The quantitative data was concerned with studying the direct relationships between performance in mathematical thinking and Mathematics Achievement of 17 year old students in Jordan. The researcher administered two tests and two interviews; a test of mathematical thinking, a test of Mathematics Achievement, teacher interviews and student interviews. More than 500 students participated in the two tests, with 13 teachers participating individually in the teacher interviews and four groups of students being involved in student interviews.

As an initial step, the relationships between the six aspects of mathematical thinking were examined using Pearson product-moment correlation coefficients. The relationships of each of the six aspects and the total score for mathematical thinking and Mathematics Achievement were also examined. The higher level of correlation between the six scales and the total for mathematical thinking is to be expected, given that the score for each aspect is included in the total score. However, the almost equally-high correlations between the six scale scores and Mathematics Achievement are of greater interest, because the assessments made were entirely independent with separate measuring instruments being used.

(Jaleel, 2015) had a study on finding the relationship between Mathematical Thinking and Achievement in Mathematics among Secondary School Students for total sample and relevant sub samples based on Gender of Students. The investigator adopted survey method for the study on a sample of 112 Secondary School Students taken at random. The relationship between Mathematical Thinking and Achievement of students at secondary level was calculated using the Pearson's Product Moment Coefficient of Correlation. The study can be concluded that there exists significant positive relationship between Mathematical Thinking and Achievement of students at secondary level for total sample. There exists no significant relationship between Mathematical Thinking and Achievement in Mathematics among Secondary School Students gender-wise.

According to the study of (Zaman, 2011) , there was an objective to find the relationship between mathematical thinking and Mathematics Achievement of the students at secondary level. The sample selected for this study was 544 students from zone 2 and 4 comprising of 10 districts of the Khyber Pakhtunkhwa province. Stratified random sampling technique was used to take the sample for this study. On the research two tools were used in the survey to collect data from students, a test of Mathematical Thinking and a test of Mathematics Achievement to collect quantitative data. These instruments were supplemented by two interviews one from students and the other from teachers to collect qualitative data. It was found the relationship between these two main variables along with relationship of individual aspect of mathematical thinking with Mathematics Achievement was determined using Pearson correlation coefficient. The correlation coefficients of mathematical thinking with Mathematics Achievement is 0.49 which significant at $p < 0.01$. All the coefficients of different aspects of Mathematical Thinking with Mathematics Achievement were significant. The correlation coefficients of the six aspects of Mathematical Thinking with the total score for Mathematical Thinking and Mathematics Achievement were also positive and statistically significant.

Generally it is said that our teachers just solve the problem on the board and say this one is the most important for the examination. So students do just rote memorization of the problems. They never enhance mathematical thinking though our curriculum alerts on developing mathematical thinking. Mathematical Thinking can be developed through mathematics teaching learning activities. Research work on this particular aspect, investigating reasoning and thinking ability and its relationship with Mathematics Achievement in Nepal is necessary. So, this study was mainly concerned on the Mathematical Thinking and Mathematics Achievement. To find relationship between the level of mathematical thinking and Mathematics Achievement was the main objective of the study.

II. Methods

This research paradigm is post positivism. This study depends on deductive approaches to find the relationship between mathematical Thinking and Mathematics Achievement. The researcher used mixed methods where both quantitative data i.e. test scores from test of Mathematical Thinking and Mathematics Achievement and qualitative data i.e. interviews from students and teachers were collected simultaneously and both types of data were adopted to address the research hypothesis. The results from both types of instruments were analyzed using both quantitative and qualitative methods. The researcher used the Multi stage sampling to find the study area. Stratified random sampling was opted because it gives representation to different strata along with minimizing sampling error. The 40 schools were selected from the study areas and 400 students 200 male and 200 female were selected as a sample from the total population. The information was collected directly from the respondents of different areas by using Mathematical Thinking Test MTT (Nepal 2015), Mathematics Achievement Test MAT (Nepal 2015) the tools that were designed as the research instruments. The relation between every aspects of Mathematical Thinking and the Mathematics Achievement was calculated using Pearson Correlation Coefficient.

III. Result And Discussion

1. Description of Students overall Achievement in all Variables of Mathematical Thinking and Mathematics Achievement

Assessment of Mathematics Achievement and mathematical thinking will be discussed in this for all students by measuring their mean score and standard deviation in all aspects of Mathematical thinking (Deduction, Induction, logical Thinking, Generalization, Mathematical Proof and Problem solving),

Mathematical thinking (Total) and Mathematics Achievement. Two types of tests one for Mathematical Thinking and another for Mathematics Achievement were administered to 400 students among 40 schools from 3 districts of Nepal. Maximum score of Mathematical thinking was 113 and Maximum score of Mathematics Achievement was 157. Average score of the test of the level of Mathematical Thinking was 37.9 and average scores of Mathematics Achievement was 41.74 calculated. Every score in each aspects of level of Mathematical thinking was calculated which was shown in the table below. The maximum mean score of all students was in deduction which was 9.85. While the minimum scores in the aspects of induction and generalization with mean score of 5.11 and 5.28 respectively.

Table 1 Description of Students overall Achievement in all Variables of Mathematical Thinking and Mathematics Achievement

Scale	Minimum	Maximum	Mean	Std. Deviation
Deduction	0	15	9.85	3.662
Induction	0	15	5.11	3.447
Logical Thinking	0	46	6.05	4.158
Generalization	0	15	5.28	3.656
Mathematical proof	0	20	5.83	4.863
Problem solving	0	20	6.06	4.900
Mathematical Thinking (Total)	0	113	37.90	18.819
Mathematics Achievement	0	157	41.74	22.656

Field survey: 2015

Table 1 shows that the mean score for induction, generalization and mathematical proof (5.11, 5.28, and 5.83) was low for all the students. It noticed that these aspects are the high level of difficulty. Similarly, The mean score for Deduction (9.85) is high. The mean score of Mathematical Thinking (Total) is 37.90 and the standard deviation of Mathematical thinking is 18.81. But the mean score of Mathematics Achievement is 41.74 and standard deviation of Mathematics Achievement is 22.66. This spread of score showed that students' mathematical thinking is quiet less than Mathematic Achievement. It also indicated the variability in different aspects in the students' abilities.

2. Relationship between Mathematical Thinking and Mathematics Achievement of grade X students.

The main objective of this study was to find the relationship between Mathematical Thinking and Mathematics Achievement. There are six aspects of Mathematical thinking and single aspects of Mathematics Achievement. The relation between every aspects of Mathematical Thinking and the Mathematics Achievement was calculated using Pearson Correlation Coefficient. The determined values are shown in the table below.

Table 2 Pearson Correlation Coefficient of aspects of Mathematical Thinking, Mathematical Thinking Total and Mathematics Achievement.

	Deduction	Induction	Logical Thinking	Generalization	Mathematical Proof	Problem Solving	MTT	MAT
Deduction	1	.412**	.394**	.376**	.376**	.320**	.404**	.612**
Induction		1	.524**	.531**	.555**	.346**	.396**	.720**
Logical Thinking			1	.575**	.610**	.497**	.614**	.798**
Generalization				1	.598**	.504**	.574**	.781**
Mathematical Proof					1	.622**	.584**	.849**
Problem Solving						1	.682**	.759**
MTT							1	.731**
MAT								1

** Correlation is significant at the 0.01 level (2-tailed).
 MTT : Mathematical Thinking Total
 MTA: Mathematical Achievement Total

Field survey: 2015

Table 2 shows that correlation coefficients of mathematical thinking with Mathematics Achievement is 0.731 which is significant at $p < 0.01$. The higher level of correlation between the six scales and the total for mathematical thinking is to be expected, given that the score for each aspect is included in the total score. It was hypothesized that the different aspects of Mathematical Thinking would be related to Mathematics Achievement to varying extents. From the table, all the coefficients of different aspects of mathematical thinking with Mathematics Achievement were significant. Deduction and Induction were the least correlated aspects to Mathematical Thinking Total being the range .404 and .396 . All six aspects of Mathematical Thinking were strongly correlated to Mathematics Achievement. Mathematical Proof, being in range .849, is the highest correlated aspect and Deduction being range 0.620 is the least correlated aspect among six. In summary, the correlations indicate that, when considered in isolation from each other, all six aspects of Mathematical Thinking were strongly related to Mathematics Achievement. Mathematical Thinking total is strongly correlated

to Mathematics Achievements being coefficient 0.731. There is strong relationship between Mathematical Thinking and Mathematics Achievements.

Among the 20 teachers they strongly agreed that there is highly correlation between Mathematical Thinking and Mathematics Achievements. Among them almost teachers said Deduction and Logical Thinking are the least difficult aspects of Mathematical Thinking and Problem Solving and Mathematical Proof are the most difficult aspects. They all agreed all the aspects of Mathematical Thinking also be correlated to the Mathematics Achievement separately too which support the quantitative result of the study.

This result is similar to the research (Ma'Moon, 2005; Zaman, 2011; Jaleel, 2015) that the strongly correlation between Mathematical thinking and Mathematics Achievement. Deduction is the least correlated among the aspects of Mathematical Thinking in the study . But In the study of (Ma'Moon, 2005) Mathematical Proof was also the most difficult aspect, while Logical Thinking was the least difficult. The study of (Zaman, 2011) showed Generalization was the easiest aspect and Mathematical Proof was the most difficult aspect.

IV. Conclusion

The study can be concluded that mean score of the students in the test of Mathematics Achievement was higher than mean score in the test of Mathematical Thinking .The score in the tests of Mathematical Thinking was significantly correlated to the score in the test of Mathematics Achievement. Individual aspects of Mathematical Thinking were all positively correlated with the score in test of mathematic achievement. All individual aspects of Mathematical Thinking were highly correlated with Mathematical Thinking (total). Problem solving sub scale was found to have the highest correlation with test of Mathematical Thinking while Induction had the lowest correlation. Among the aspects, Mathematical Proof was found to be highly correlated Mathematics Achievement while the lowest correlation was deduction with test of Mathematics Achievement. In summary, all the aspects of Mathematical Thinking and Mathematical Thinking (total) are correlated with Mathematics Achievement.

From above findings of the study, it can be suggested that it is necessary for students to enhance their Mathematical Thinking. Teachers should focus on developing the aspects of Mathematical Thinking through pedagogical instructional activities and content or knowledge of these aspects that help to increase the standard of Mathematics Achievement. Teacher, Trainer will be concerned to run training package and conduct Mathematics teachers' training to develop the students' mathematical Thinking. It will be fruitful to enhance the level of the aspects of Mathematical Thinking. The book authors should pay attention to compose their matters on the way of developing the aspects of Mathematical Thinking. It is also recommended for further research in order to fill up the research gaps on this study.

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